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MEMORANDUM REPORT No. 880

Aerodynamic Derivatives For Both Steady And Non-Steady Motion Of Slender Bodies

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DEPARTMENT OF THE ARMY PROJECT No. 503-03-001

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BALLISTIC RESEARCH LABORATORIES

ABERDEEN PROVING GROUND, MARYLAND

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EMWood/CHMurphy/jmc
Aberdeen Proving Ground, Md.
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ABSTRACT

Slender body values of all non-Magnus first order aerodynamic coefficients are derived by a simple application of the concept of cross-sectional apparent mass. These results are converted to ballistic nomenclature for two types of motion.

INTRODUCTION

The application of Munk's slender body theory to the calculation of aerodynamic coefficients describing steady motion (α, q constant) has been made by a number of authors[1, 2, 3]. Dorrance[4] has obtained slender body values of those coefficients associated with non-steady angles of attack as a by-product by a linear analysis of the potential equation for non-steady supersonic flow. The stability derivatives for the most general motion of slender bodies with arbitrary cross-section have been calculated by Sacks[5]. This treatment is, however, quite complex. This paper will present a simple derivation of all non-zero stability derivatives of a slender body of revolution.

CALCULATION OF SLENDER BODY COEFFICIENTS

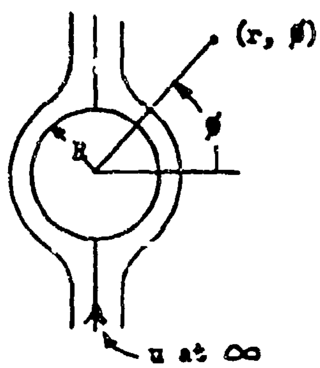

FIG. 1

We shall make the usual assumption that the cross flow, u, at each cross-section is a two-dimensional potential flow, and independent of both the axial flow and the cross flows at neighboring cross-sections. The flow, as seen from the body (Fig. 1) is the potential of a steady flow plus a doublet, $u(r + R^2/r) \sin \phi$, where r and ϕ are polar coordinates, $R = R(x)$ is the radius of the cross-section and x is its distance from the base of the missile. The potential of the flow with respect to stationary air can be obtained by subtracting the potential of the steady flow u and is

$$\psi = u\,(R^2/r) \sin \phi\,. \tag{1}$$

In Ref. 6 it is shown that the kinetic energy of an infinite volume V of fluid containing a body with surface area S is

$$\frac{\rho}{2}\int_V (\nabla\psi)^2\, dV = -\frac{\rho}{2}\int_S \psi\,\frac{\partial\psi}{\partial n}\, dS, \tag{2}$$

where ρ is the density and $\partial/\partial n$ is the derivative with respect to the outward pointing normal. The kinetic energy of the fluid per unit length due to the presence of the circular cross-section, therefore, is, from Eqs.(1) and (2),

$$(-\rho/2)\int_0^{2\pi} (uR \sin \phi)\ (-u \sin \phi)R\ d\phi = (u^2/2)(\rho\pi R^2)\ . \quad (3)$$

Eq. (3) has the form of $\frac{u^2}{2}$ multiplied by an "apparent mass". Thus the apparent mass per unit length of each cross-section is ρA, where A is its area, and hence

$$\text{Momentum of the fluid per unit length} = \rho A u\ . \quad (4)$$

If we consider the momentum of the fluid at adjacent cross-sections we see that it will depend on the local area and fluid velocity. Now the axial velocity, U, will move the fluid from one cross-section to the next and hence from a region of higher momentum to one of lower momentum or vice versa. This local change in momentum gives rise to a differential force which can then be integrated to give the total force on the body. Since we are then actually interested in the location of the cross-section of fluid and its motion, we will interpret x to be its location with respect to the base.

We shall make use of a non-rolling right-handed coordinate system $\tilde{X}\tilde{Y}\tilde{Z}$ with $\tilde{X}$ pointing forward along the missile's axis and $\tilde{Y}$ axis initially in the horizontal plane.* A non-rolling coordinate system will mean one whose x-component of angular velocity is zero. As is shown in Ref. 7, the treatment of time derivatives of the dynamic variables i.e., $\dot{\alpha}$, $\dot{q}$, is much more natural in non-rolling coordinates.**

If we consider projections of the velocity vector and a fixed direction vector on the $\tilde{X}\ \tilde{Z}$ plane, and with α and θ as defined in Fig. 2,

$$u = -U\ \alpha + (x - x_{cg})q, \quad (5)$$

* We make use of the tilde superscript to distinguish these axes from the standard aerodynamic missile-fixed axes. In the case of a spin-stabilized body of revolution, those axes would be spinning rapidly with respect to our non-rolling axes, and hence, with respect to the air at infinity.

** In Ref. 5 Sacks uses a missile fixed system but specifically measures $\dot{\alpha}$ and $\dot{\beta}$ in a non-rolling system. However he does measure $\dot{q}$ and $\dot{r}$ in the rolling system and therefore obtains non-zero values of $C_{L_{pr}}$ and $C_{m_{pr}}$ for a body of revolution.

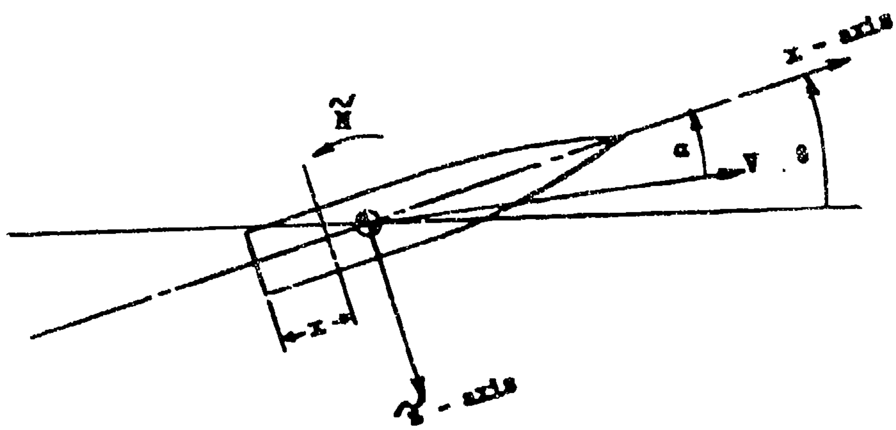

FIG. 2

where $U = -\frac{dx}{dt}$ is the axial component of the missile's velocity, x_{cg} is distance of center of gravity from base and x is measured from the base, and $q = \dot{\theta}$.

By Newton's Second Law and Eqs. (4, 5) the force per unit length in the $\tilde{Z}$ direction, $\frac{d\tilde{Z}}{dx}$, is the time rate of change of momentum per unit length.*

$$\therefore \frac{d\tilde{Z}}{dx} = \frac{d}{dt}(\rho A u)$$

$$= \rho U^2 \left[\alpha \frac{dA}{dx} - \frac{\dot{\alpha}}{U} A - \frac{q}{U}\left[A + (x - x_{cg})\frac{dA}{dx}\right] + \frac{\dot{q}}{U^2}(x - x_{cg})A\right] \quad (6)$$

Integrating over the length of the missile,

$$\tilde{Z} = \rho U^2 \left\{-\alpha S_b - \frac{\dot{\alpha}}{U} v d^3 - \frac{q}{U} S_b x_{cg} + \frac{\dot{q}}{U^2}(x_c - x_{cg}) v d^3\right\}, \quad (7)$$

* Since it is actually assumed that the total velocity V is constant, the derivative of U, the axial component, should appear in Eq. 6. Fortunately, for small α and $\dot{\alpha}$ this quantity is small and can be neglected.

where $\int_0^L \frac{dA}{dx}\, dx = - S_b,$ (base area)

$\int_0^L A dx = v d^3,$ (v is dimensionless volume)

$\int_0^L (x - x_{cg}) A dx = (x_c - x_{cg}) v d^3,$ (x_c is x-coordinate of centroid)

and d is diameter.

If we multiply $\frac{d\tilde{Z}}{dx}$ by $-(x - x_{cg})$ and integrate,* we can obtain the moment about the c.g.

$$
\begin{aligned}
\tilde{M} &= -\int_0^L \frac{d\tilde{Z}}{dx}\,(x - x_{cg})\,dx \\
&= \rho U^2 \left\{ \alpha \left[v d^3 - x_{cg} S_b \right] + \frac{\dot{\alpha}}{U} \left[v d^3 (x_c - x_{cg}) \right] \right. \\
&\quad - \frac{q}{U} \left[x_{cg}{}^2 S_b + (x_c - x_{cg}) v d^3 \right] \\
&\quad \left. + \frac{\dot{q}}{U^2} \left(\bar{k}_b{}^2 d^2 - 2 x_c x_{cg} + x_{cg}{}^2 \right) v d^3 \right\} ,
\end{aligned}
\qquad (8)
$$

where $\int_0^L (x - x_{cg})^2 A dx = v \bar{k}^2 d^5$

$$= v(\bar{k}_b{}^2 d^2 - 2x_c x_{cg} + x_{cg}^2) d^3 ;$$

$\bar{k}$ is the dimensionless transverse radius of gyration of a homogeneous model about its center of gravity, and

$\bar{k}_b$ is the dimensionless transverse radius of gyration of a homogeneous model about its base.

* Fig. 2 indicates the direction of positive $\tilde{M}$.

An examination of Eqs. (7) and (8) shows that, under our assumptions, $\tilde{Z}$ and $\tilde{M}$ depend linearly on α, $\dot{\alpha}$, q, and $\dot{q}$. In order to obtain the relations for the various stability derivatives we first write the equations defining them.

$$\tilde{Z} = \frac{\rho V^2}{2} S \left\{ C_{\tilde{Z}_\alpha} \alpha + C_{\tilde{Z}_{\dot{\alpha}}} \frac{\dot{\alpha} d}{2V} + C_{\tilde{Z}_q} \frac{q d}{2V} + C_{\tilde{Z}_{\dot{q}}} \frac{\dot{q} d^2}{4V^2} \right\} , \qquad (9)$$

$$\tilde{M} = \frac{\rho V^2}{2} Sd \left\{ C_{\tilde{M}_\alpha} \alpha + C_{\tilde{M}_{\dot{\alpha}}} \frac{\dot{\alpha} d}{2V} + C_{\tilde{M}_q} \frac{q d}{2V} + C_{\tilde{M}_{\dot{q}}} \frac{\dot{q} d^2}{4V^2} \right\} , \qquad (10)$$

where S is reference area and V is total velocity.

If we compare Eqs. (7) and (8) with Eqs. (9) and (10), introduce the non-dimensional $\hat{x} = \frac{x}{d}$, and remember that for small angles of attack and yaw U can be replaced by V, we can observe that the following relations are true.

$$C_{\tilde{Z}_\alpha} = -2 \frac{S_b}{S}, \qquad C_{\tilde{M}_\alpha} = \left(-2 \frac{S_b}{S}\right) \hat{x}_{cg} + \frac{2v}{S/d^2} ,$$

$$C_{\tilde{Z}_{\dot{\alpha}}} = -4 \frac{v}{S/d^2}, \qquad C_{\tilde{M}_{\dot{\alpha}}} = \left(-4 \frac{v}{S/d^2}\right)(\hat{x}_{cg} - \hat{x}_c), \qquad (11)$$

$$C_{\tilde{Z}_q} = \left(-4 \frac{S_b}{S}\right)\hat{x}_{cg}, \qquad C_{\tilde{M}_q} = \left(-4 \frac{S_b}{S}\right)\hat{x}_{cg}^2 + \left(\frac{4v}{S/d^2}\right)(\hat{x}_{cg} - \hat{x}_c),$$

$$C_{\tilde{Z}_{\dot{q}}} = \left(-8 \frac{v}{S/d^2}\right)(\hat{x}_{cg} - \hat{x}_c), \qquad C_{\tilde{M}_{\dot{q}}} = -\frac{8v}{S/d^2} \hat{k}^2 = \frac{-8v}{S/d^2}(\hat{x}_{cg}^2 - 2\hat{x}_c \hat{x}_{cg} + \hat{k}_b^2) .$$

The corresponding relations for $\tilde{Y}$ and $\tilde{N}$ can be found from symmetry considerations. It can easily be shown[8] that:

$$C_{\tilde{Y}_\beta} = C_{\tilde{Z}_\alpha}, \qquad C_{\tilde{N}_\beta} = -C_{\tilde{M}_\alpha}$$

$$C_{\tilde{Y}_r} = -C_{\tilde{Z}_q}, \qquad C_{\tilde{N}_r} = C_{\tilde{M}_q},$$

$$C_{\tilde{Y}_{\dot{\beta}}} = C_{\tilde{Z}_{\dot{\alpha}}}, \qquad C_{\tilde{N}_{\dot{\beta}}} = -C_{\tilde{M}_{\dot{\alpha}}}, \qquad (12)$$

$$C_{\tilde{Y}_{\dot{r}}} = -C_{\tilde{Z}_{\dot{q}}}, \qquad C_{\tilde{N}_{\dot{r}}} = C_{\tilde{M}_{\dot{q}}} .$$

Since the derivation of Eqs. (11) was based on the assumption of non-viscous flow, this theory makes no prediction of Magnus force and moment derivatives. We see, however, that Munk's elementary theory based on the concept of the apparent mass of each cross-section will provide slender body values of all sixteen first order non-Magnus aerodynamic coefficients.

CONVERSION OF RESULTS TO BALLISTIC NOMENCLATURE

In order to state our results in ballistic nomenclature[9] we first write the definition of the non-Magnus ballistic K's in terms of the complex quantities $\tilde{Y} + i\tilde{Z}$, $\tilde{M} + i\tilde{N}$, $p + is$, $q + ir$, $\beta + i\alpha$ and $\dot{q} + i\dot{r}$ and make use of the relation $U \doteq V$. (This use of the complex variable exploits the rotational symmetry of the usual configurations studied by ballisticians).

$$\tilde{Y} + i\tilde{Z} = \rho V^2 d^2 \left[- K_N(\beta + i\alpha) + iK_S \left(\frac{(q + ir)d}{V}\right)\right] ,$$

$$\tilde{M} + i\tilde{N} = \rho V^2 d^3 \left[- iK_N(\beta + i\alpha) - K_H \left(\frac{(q + ir)d}{V}\right)\right] . \tag{13}$$

The corresponding expressions for the aerodynamic C's are:*

$$\tilde{Y} + i\tilde{Z} = \left(\frac{1}{2}\rho V^2\right)S \left[C_{\tilde{Z}_\alpha} (\beta + i\alpha) + iC_{\tilde{Z}_q}\left(\frac{(q + ir)d}{2V}\right) + C_{\tilde{Z}_{\dot{\alpha}}} \left(\frac{(\dot{\beta} + i\dot{\alpha})d}{2V}\right) + iC_{\tilde{Z}_{\dot{q}}}\left(\frac{(\dot{q} + i\dot{r})d^2}{4V^2}\right)\right] , \tag{14}$$

$$\tilde{M} + i\tilde{N} = \left(\frac{1}{2}\rho V^2\right)Sd \left[- iC_{\tilde{m}_\alpha}(\beta + i\alpha) + C_{\tilde{m}_q} \left(\frac{(q + ir)d}{2V}\right) - iC_{\tilde{m}_{\dot{\alpha}}} \left(\frac{(\dot{\beta} + i\dot{\alpha})d}{2V}\right) + C_{\tilde{m}_{\dot{q}}} \left(\frac{(\dot{q} + i\dot{r})d^2}{4V^2}\right)\right] .$$

Since there is not a one-to-one correspondence between the ballistic and aerodynamic coefficients, the best we can do is to state relations between them for specified motions. Angular motion with respect to the trajectory of a spin-stabilized body of revolution consists of a transient oscillatory motion and a steady state "yaw of repose". The trajectory can be considered essentially straight over a reasonable number of periods of the transient motion and hence for this motion, $\dot{\alpha} = q$ and $\dot{\beta} = - r$.

$$\therefore \tilde{Y} + i\tilde{Z} = \left(\frac{1}{2}\rho V^2\right) S \left[C_{\tilde{Z}_\alpha} (\beta + i\alpha) + i(C_{\tilde{Z}_q} + C_{\tilde{Z}_{\dot{\alpha}}})\left(\frac{(q + ir)d}{2V}\right) + C_{\tilde{Z}_{\dot{q}}} \left(\frac{(\ddot{\beta} + i\ddot{\alpha})d^2}{4V^2}\right)\right] , \tag{15}$$

$$\tilde{M} + i\tilde{N} = \left(\frac{1}{2}\rho V^2\right) Sd \left[- iC_{\tilde{m}_\alpha} (\beta + i\alpha) + (C_{\tilde{m}_q} + C_{\tilde{m}_{\dot{\alpha}}})\left(\frac{(q+ir)d}{2V}\right) - iC_{\tilde{m}_{\dot{q}}} \left(\frac{(\ddot{\beta} + i\ddot{\alpha})d^2}{4V^2}\right)\right] .$$

* It should be emphasized again that the axes form a non-rotating system. The conversion from the rotating to the non-rotating system is given in Ref. 7.

In the Appendix it is shown that for large ratios of mass of missile to mass of displaced fluid the angular acceleration terms involving $\ddot{\beta}$ and $\ddot{\alpha}$ in Eqs. (15) may be neglected. The situation is different for an airship or a torpedo for which this ratio is of order unity and these terms are important.

$\therefore$ for transient motion about a straight trajectory, from Eqs.(11), (13), and(15):

$$
\begin{aligned}
K_N &= -\frac{1}{2}\frac{S}{d^2} C^{\sim}_{Z\alpha} = a_b \,, \\
K_S &= \frac{1}{4}\frac{S}{d^2} (C^{\sim}_{Zq} + C^{\sim}_{Z\dot{\alpha}}) = -a_b \hat{x}_{cg} - v \,, \\
K_M &= \frac{1}{2}\frac{S}{d^2} (C^{\sim}_{m\alpha}) = -a_b \hat{x}_{cg} + v \,, \\
K_H &= -\frac{1}{4}\frac{S}{d^2} (C^{\sim}_{mq} + C^{\sim}_{m\dot{\alpha}}) = a_b \hat{x}_{cg}^{\,2} \,,
\end{aligned}
\tag{16}
$$

where $a_b = \dfrac{S_b}{d^2}$ = base area in calibers squared.

For the steady motion of the yaw of repose $\dot{\alpha} = \dot{\beta} = 0$, and it can be shown that the $\dot{q} + i\dot{r}$ contributions are small.

$$
\begin{aligned}
\therefore\quad K_N &= -\frac{1}{2}\frac{S}{d^2} C^{\sim}_{Z\alpha} = a_b \,, \\
K_S &= \frac{1}{4}\frac{S}{d^2} C^{\sim}_{Zq} = -a_b \hat{x}_{cg} \,, \\
K_M &= \frac{1}{2}\frac{S}{d^2} C^{\sim}_{m\alpha} = -a_b \hat{x}_{cg} + v \,, \\
K_H &= -\frac{1}{4}\frac{S}{d^2} C^{\sim}_{mq} = a_b \hat{x}_{cg}^2 - v(\hat{x}_{cg} - \hat{x}_c).
\end{aligned}
\tag{17}
$$

It is quite interesting to note that the absence of a one-to-one correspondence between the aerodynamic and the ballistic equations leads to coefficients, which, in the ballistic notation, depend in general on the type of motion under consideration. This result is inherent in the definitions of the coefficients. This difficulty is resolved in a report being prepared by one of the authors[7].

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APPENDIX

In order to estimate the magnitude of the $(\ddot{\alpha} + i\ddot{\beta})$ terms we have to analyze the transient motion a little more.* α and β are the sums of the two sinusoidal motions** differing in phase by 90° and with frequencies $\dot{\phi}_1$ and $\dot{\phi}_2$ where

$$\dot{\phi}_{1,2} = \frac{V}{d}\sqrt{\frac{1}{2}\,\frac{\rho S d}{m}\,k_t^{-2}}\left\{(C_{\widetilde{m\alpha}}\, s)^{1/2} \pm \left[C_{\widetilde{m\alpha}}(s-1)\right]^{1/2}\right\}, \quad \left(\frac{1}{s} < 1\right) \qquad \text{(A-1)}$$

where m is mass of missile,

k_t is transverse radius of gyration in calibers,

$$s = \frac{k_a^4\, k_t^{-2}\left(p\,\frac{d}{V}\right)^2}{2\left(\rho S\,\frac{d}{m}\right) C_{\widetilde{m\alpha}}} \quad \text{(ballistic stability factor)},$$

k_a is axial radius of gyration in calibers,

p is axial spin.

For any sinusoidal motion we have that

$$\begin{aligned} \ddot{\beta} &= -\dot{\phi}_1^2\, \beta\,, \\ \ddot{\alpha} &= -\dot{\phi}_1^2\, \alpha\,, \end{aligned} \qquad \text{(A-2)}$$

or
$$\ddot{\beta} + i\ddot{\alpha} = -\dot{\phi}_1^2\,(\beta + i\alpha)\,.$$

In order to get a size estimate we consider only the contribution from the fast rate $\dot{\phi}_1$.

$$\therefore\ \frac{(\ddot{\beta} + i\ddot{\alpha})d^2}{V^2} = (\beta + i\alpha)\left(\frac{1}{2}\,\frac{\rho S d}{m}\,k_t^{-2}\right)\left\{(C_{\widetilde{m\alpha}}\, s)^{1/2} + \left[C_{\widetilde{m\alpha}}(s-1)\right]^{1/2}\right\}^2 . \qquad \text{(A-3)}$$

We are interested in the relative contribution to the force or moment of the term in $\ddot{\alpha} + i\ddot{\beta}$ and the term involving $\alpha + i\beta$, in Eqs.(15). Since $C_{\widetilde{m\alpha}}$ is the same order of magnitude as $C_{\widetilde{m\dot{q}}}$, (which can be seen from Eqs. 11),

---

* The following analysis is valid for any statically or gyroscopically stable missile. The special case of a statically stable missile with exactly zero spin can be considered as a limiting case of a statically stable missile with small spin; therefore, $1/s = -\infty$

** See Reference 9 for a discussion of Eq. (A-1).

then the ratio of the acceleration term to the linear term is

$$\frac{\text{term in } \ddot{\beta} + i\ddot{\alpha}}{\text{term in } \beta + i\alpha} = \left(\frac{\dot{\phi}_1 d}{2V}\right)^2 = \frac{1}{8}\,\frac{\rho S d}{m}\,k_t^{-2}\left\{\left(C_{M_\alpha} s\right)^{1/2} + \left[C_{M_\alpha}(s-1)\right]^{1/2}\right\}^2 . \quad (A\text{-}4)$$

Note that $\rho Sd/m = (m_r/m)\,(Sd/V)$, where V/Sd is the length in calibers of the equivalent cylindrical volume, m_r is the mass of displaced fluid and m is the mass of the missile. We now state bounds for various quantities, for practical configurations:

<u>gyroscopically stable</u>	<u>statically stable</u>
$k_t^{-2} < 1$	$k_t^{-2} < 1$
$1 < s \leq 6$	$-6 \leq s < 0$
$0 < C_{M_\alpha} \leq 10$	$-10 \leq C_{M_\alpha} < 0$
$V/Sd > 3$	$V/Sd > 3$

Thus,
$$\frac{\text{term in } \ddot{\alpha} + i\ddot{\beta}}{\text{term in } \alpha + i\beta} < \frac{m_r}{m}\left(\frac{1}{8}\right)\left(\frac{1}{3}\right)(1)\left(\sqrt{60} + \sqrt{70}\right)^2$$

$$< 12\, m_r/m \quad \text{in either case.}$$

For the usual missile in air, this ratio is negligibly small and consequently these acceleration terms are usually neglected. For an airship, or where the fluid is water (e.g., torpedoes), the acceleration terms may become appreciable.

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